Dialogical and Narrative Perspectives on Online Collaborative Mathematics Problem Solving

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Overview

- The **Virtual Math Teams** (VMT) project at The Math Forum.
- Narrative Learning Environments **and** Computer-supported Collaborative Problem Solving:
  - Active exploration
  - **Co-construction**
  - Reflection
- Dialog and narration
- Implications for **design**

The Virtual Math Teams (VMT) Project @ The Math Forum

- An **interdisciplinary** research program.
- Investigates innovative use of online collaborative environments to **support** effective K-12 mathematics learning:
  - **Talk-in-interaction** and its relationship with understanding mathematics.
  - **Pedagogical** issues.
  - **Design** issues.
  - **Methodological** issues

Virtual Math Teams: PowWows

- Extend the Math Forum's "Problem of the Week (PoW)"
  - Groups of 3 to 5 students
  - Grades 6th to 11th
  - 60-90 minutes
  - Non-routine mathematical problems
- Study I: minimal support with AOL instant messenger.
- Data source: Chat logs
- Mixed methods analysis (Statistical Modeling and Conversation Analysis).
- Case Study: Emergence of narrative structures
  - Type of problem: Word/Story Problem ("*Pressed for Time*")
  - 3 main co-particiapnts: SKI, YAG and GOH

Narrative Learning Environments

- R&D in Narrative Learning Environments (NLE) explores intelligent learning environments where “narrative is approached and applied” to support learning and the **construction of meaning.**

- NLEs promote three main kinds of activities for learners:
  - ***Co-construction***: the ability to participate in the construction of a narrative.
  - Active ***exploration*** of the learning tasks: trying to understand and reason about a narrative environment and its elements.
  - ***Reflection***: consequent sense-making of what happened within the learning activities.

The *Dialogic* Perspective

- Points to the features of **talk as action**, and social action, as core processes of **human sense-making**.
- Meaning-making is viewed as an **interactional** achievement of co-participants.
- Participation and engagement are central to the learning processes conceived as a socio-cultural practice.
- Vygotsky's developmental hypothesis: "**higher-order mental functions appears twice, first in the intersubjective level and are then internalized at the intrasubjective level.**"

M.M.Bakhtin's dialogism

- Everything is a dialog (M.M.B.)
- **Utterances** should be the unit of analysis (M.M.B.)
- A deconstructivist theory, opposed to Saussure
- Learning as mastering a **speech genre** (e.g. mathematics) through and for both
  - Narrative
  - Dialog

"Pressed for Time"

The Rational Reader, a popular daily newspaper, has to be printed by 5 a.m. so that it can be distributed.

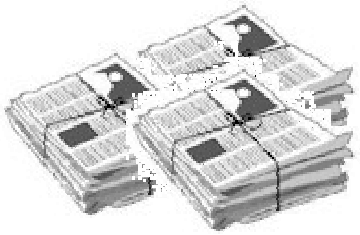
Late one night, a major story broke and the front page had to be rewritten, which delayed the start of the printing process until 3 a.m.

To try to get the printing done on time, the Reader used both their new printing press and their old one. The new press is three times as fast as the old one, and with both of them running, the printing was finished exactly on time.

How long does it take to print a normal edition of the paper using only the new press?

$[1/t_{new}] + [1/t_{old}] = [1/t_{new\&old}]$

$3^* t_{new} = t_{old}$



Context:

…

136.	7:25:46	GOH	hi. I just got here.
137.	7:25:50	SKI	o hi
138.	7:25:55	SKI	want me to xplain
139.	7:25:59	SKI	my way again?
140.	7:26:01	GOH	plz
141.	7:26:03	SKI	ok
142.	7:26:09	YAG	hi

…

Questions: How is "doing explaining" achieved? What is it that is about to be explained? (e.g. an answer? a solution strategy? a "way"?) What roles are performed in the doing of the explanation? What frameworks of participation are recognizable as relevant to the participants? Does this matter for learning/understanding? How could design be informed by these data?

144.	7:26:10	SKI	i started and solved with a system
145.	7:26:12	SKI	of equations
146.	7:26:14	YAG	let SKI explain...
147.	7:26:24	SKI	lets just say x is the time for the old machine and y is for the new
148.	7:26:29	GOH	ok
149.	7:26:35	SKI	our first equation is like this
150.	7:26:41	SKI	if we atke the recip of x
151.	7:26:45	YAG	*choughSHOWOFFchough*
152.	7:26:55	YAG	:P
153.	7:26:57	YAG	:-D
154.	7:26:59	SKI	thats how much of the job the old one does in one hour
155.	7:27:02	YAG	yep
156.	7:27:12	SKI	and the reciprocal of y is how much of the job the new one does in one hour
157.	7:27:16	YAG	recip y is the new one
158.	7:27:24	SKI	ok
159.	7:27:29	SKI	recip=reciprocal
160.	7:27:33	SKI	anyways
161.	7:27:38	YAG	and, recip y+ recip x = 1/2
162.	7:27:43	SKI	we add 1/x and 1/y
163.	7:27:48	SKI	ya
164.	7:27:50	SKI	what YAG said
165.	7:27:53	SKI	1/2
166.	7:27:56	YAG	in hours and fraction of work
167.	7:28:04	YAG	needed to be done
168.	7:28:05	SKI	cuz they together get half the job done in one hour
169.	7:28:09	YAG	:P
170.	7:28:13	SKI	are u getting our first equation?

201.	7:29:38	GOH	how come 1/x and 1/y added equal 1/2?
202.	7:29:42	SKI	ok
203.	7:29:47	YAG	ummm
204.	7:29:50	YAG	pure luck!
205.	7:29:51	SKI	1/x is how much the old one does in one hour
206.	7:29:57	GOH	right.
207.	7:29:58	SKI	how much of the job it does in an hour
208.	7:30:01	YAG	(frac of job done)
209.	7:30:03	SKI	1/y is for the new machine
210.	7:30:08	GOH	right
211.	7:30:11	SKI	add those up
212.	7:30:18	YAG	and since they do it together at 3-5
213.	7:30:20	SKI	thats how much of the job they do together in one hour
214.	7:30:22	YAG	it took 2 hrs
215.	7:30:25	SKI	ya
216.	7:30:29	SKI	listen to [YAG]
217.	...		
228.	7:31:06	SKI	the whole job took 2 hours
229.	7:31:14	YAG	with both machines
230.	7:31:19	SKI	so in one hour they did 1/2 of the job
231.	7:31:34	YAG	and in the 2nd hour they did the other half
232.	7:31:54	GOH	Okay, I got it. 1/2 is how much of the job they do together in one hour
233.	7:31:58	SKI	rite
234.	7:32:00	YAG	yepyepyep
235.	7:32:06	SKI	u know what x and y represent rite?
236.	...		

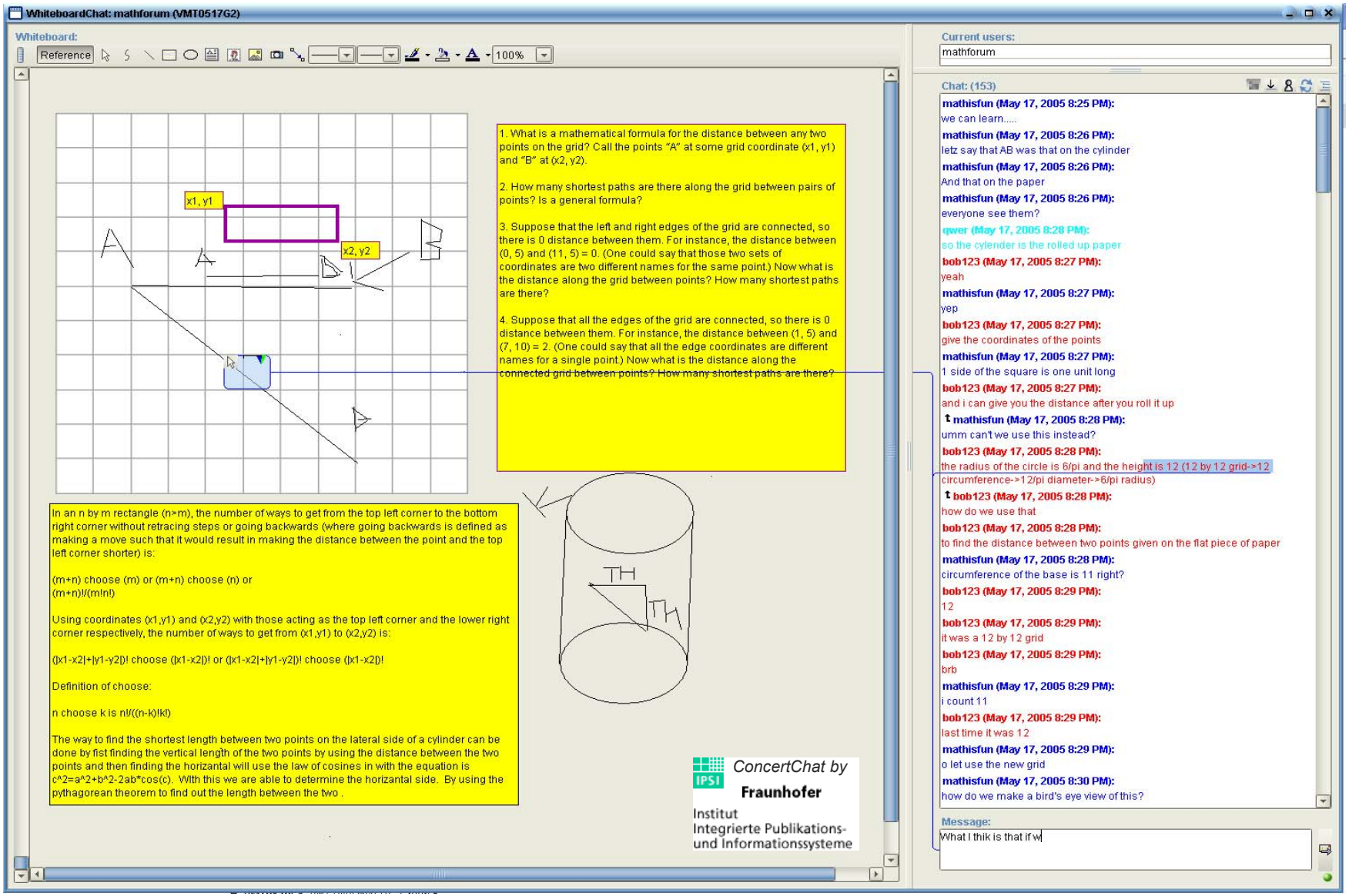
Observations:

- In this kind of participatory or interactive narrative the "*narrator*" and "*narratee*" roles are **constituted** through the ways participants orient themselves towards **participation** (e.g. producing/receiving an explanation)
- The "narrative" that emerges from interaction is **co-constructed** by both narrator and narratee.
- **Intelligibility** and "possible mathematical worlds" are two of the forces that govern the dynamics of the unfolding of the narrative/explanation.
- In summary, we propose to **connect** the ideas of narrative learning environments and collaborative learning environments by virtue of their common concern for the **role of discourse** and interaction in learning and its commitment to supporting meaning-making via **designed artefacts**.

Implications and Goals for Design

- ***Co-construction***:
  - Ameliorate breakdowns in interaction (e.g. chat confusion)
  - Scaffold problem-solving processes (e.g. capture problem solving proposals)
  - Support shared and persistent artefacts
  - "Bridge" of synchronous and asynchronous interactions.
- **Active *Exploration:***
  - Support rich and purposeful individual, small-group, and community interactions (e.g. integrate whiteboard support)
  - Allow flexible use of resources (e.g. referencing).
- ***Reflection***:
  - Provide records of interactions as resources for participants to access their own activity
  - Investigate automated narrative summarization and intelligent indexing to facilitate re-usability and "bridging" of interactions
  - Support knowledge building in an online community of math problem-solvers.

An Example: Fraunhofer IPSI *ConcertChat*



Expanding the scope of NLEs

- Participatory or interactive narratives offer opportunities for co-construction of meaning precisely based on the **dialogic principle of interactivity** resulting on an intermix of "narrative structures" (e.g. narrator, events, point of view, etc.) and the frameworks of shared participation observable in records of interaction.

- We seek to illustrate this observation within the domain of online, collaborative **mathematical problem solving**.

- In summary, we propose to **connect** narrative learning environments and collaborative learning environments by virtue of their common concern for the **role of discourse** and interaction in learning and its commitment to supporting meaning-making via **designed artefacts**.

[From 'Riz', Math Forum POW-Mentor]

I think this problem was a little scary looking, as the number of solutions submitted was much lower than usual. It was a tough problem, but it was very interesting to look over the solutions that did come in, because there were lots of different ways that people thought about and approached solving it. Let's take a look at the solutions shown below.

Andy solved the problem by reasoning it out in terms of the times involved. His key insight, which others had as well, was that with the new press being 3 times faster than the old one, the job of printing the paper with both presses was 3/4 finished by the new one and 1/4 finished by the old one. Arthur also used that thinking and created a proportion to solve the problem.

Sasha did a nice job of using variables to represent the papers per hour that each press can print, then created equations to determine how long it would take each press by itself.

[From 'Andy', 16, Concord High School, DE]

Using just the new press, it take 2 hours and 40 minutes to print an edition of the newspaper.

On the night of the breaking news it took (2) hours to print the newspaper using both the old and new press, from 3am to 5am. The new press is 3 times as fast the old one. Meaning that it printed 75% of the total papers that evening. In order to determine the amount of time it would take to print the newspaper using the new press alone, I used a proportion. The new press was able to print 75% of the papers in 2 hours, and it would be able to print 100% of the papers in x hours. This proportion would look like:

75% = 2hours
100% x hours

This could be reduced to:

3 = 2
4 x

In this proportion we are trying to find x which would be the amount of time required to print an edition of the paper using the new printing press alone. Using cross multiplication...

[From Sasha(12), John Glenn Middle School, MA]

The new press takes two hours and forty minutes to print a regular copy.

First, I have to define variables for my objects in the problem. Let b be the rate of the old printing press, in papers printed per hour. Let a be the rate of the new printing press, in papers printed per hour. Let p be the total number of papers printed.

So, now I have to figure out how I am doing this problem. In two hours, the problem says both the new press and the old press together print the right amount of copies, p. So, we can put this into equation form like this: 2a + 2b = p

Instead of having a + b = p, I had 2a and 2b. This is because both machines were working for two hours. So, since the variables a and b stand for the papers printed per hour, if you multiply it by two it will become the papers printed in two hours. Both of the printing presses added...