From **Proceedings of the Romanian Academy, Series A, Mathematics, Physics, Technical Sciences, Information Science, Vol.1, No.2, 2000.**

CATEGORIES AND FUNCTORS FOR THE STRUCTURAL-PHENOMENOLOGICAL MODELING

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A possible extension of the theory of categories to the structural-phenomenological domains of science is presented. Notions like phenomenological categories with phenomenological objects and morphisms and structural-phenomenological categories with objects formed both of structural and phenomenological parts are introduced. Concerning the functors, the most important are those between structural and phenomenological categories. The morphisms and the functors for the structural-phenomenological domains are also physical and informational processes, having a role in the reality of nature. In general, the physical and informational feasibility condition is essential for the adaptation and development of the theory of category into a mathematical structural-phenomenological theory of categories.

1. INTRODUCTION

Saunders Mac Lane and S. Eilenberg used the notions of categories and functors for the first time in 1945. [1].
The category is a sort of mathematical universe that has brought about a remarkable unification and simplification of mathematics [2].
The theory of categories is seen today as a convenient and generalized language of mathematics. But because the concept of a category is so general, it is to be expected that theorems provable for all categories will not usually be very deep. Consequently, many theorems of category theory are stated and proved for particular classes of categories? [3].

If in the domain of structural science the theory of categories brings some simplicity and elegance, yet the entire mathematics can be expressed only by the theory of sets, as it was observed by W.V.Quine (apud [4], p.221, rom.ed.). But the situation may be quite different for the structural- phenomenological science [5][6]; in this case the theory of categories might become indeed the appropriate language of description. In this paper is shown that adapting the theory of categories for the description of structural-phenomenological domains of reality is possible.
Recently, G.Kato and D.C.Struppa [7] and D.C. Struppa et al [8] considered the use of the theory of categories for both physics and the science of consciousness:
"Category theory as a generalized language of mathematics has been shown to be widely applicable to physical theories and as such it is not just a powerful mathematical language, it is

also perhaps the foundation of physical theories. Since consciousness may be difficult, if not impossible, to formulate in terms of analytical methods, a powerful, "pre-analytical" mathematical language may be appropriate for consciousness as it is for physics" [8].

In a first period the structural-phenomenological modeling used the theory of sets and automata theory together with symbols for non-formal functions and processes [9] [10].
A second period for the structural-phenomenological modeling seems to be more efficient by using categories and functors. Structural-phenomenological theories may be "detailed theories" or "envelope theories" [6]). The considerations that follow are exposed in the frame of envelope theories.

2. PHENOMENOLOGICAL CATEGORIES

One cannot speak about the theory of consciousness without taking into account the mental senses (qualia, "experience") which are phenomenological information (phenomenological senses). Also, one cannot speak about a deep physical theory without phenomenological information or phenomenological senses, these two terms being equivalent.
Because in the definition of a category, it is not required that its objects should be sets with elements [11], that is usual mathematical objects, a category with its objects being phenomenological senses is called phenomenological category.
A collection of phenomenological senses is a category if there exists morphisms among these objects, and if the composition map of morphisms and the identity morphism, like for any category [11], are respected.
A collection (family) of phenomenological senses cannot be a set if there are morphisms among its objects.
A collection of phenomenological senses is a set if among these objects there are no morphisms. But such sets of phenomenological senses could be objects of a category.

Therefore two types of phenomenological categories may be envisaged:

- a collection of phenomenological senses with morphisms among them, composition maps and identity morphism;
- a collection of sets of phenomenological senses (each such set being an object of the category), also with morphisms, composition maps and identity morphism.

The objects of a phenomenological category will be named phenomenological objects, and the morhisms, correspondingly, phenomenological morphisms.
The senses of a set of phenomenological senses (sometimes may be named phenomenological set) will be called phenomenological elements.
The morphisms of a phenomenological category are called phenomenological morphisms.
The phenomenological senses may be conceived, in an abstract way, as intrinsic non-structural objects or elements.
The phenomenological morphism may be conceived, in an abstract way, as a non-structural, non-formal, process of transformation from one phenomenological sense to another.
The categories for the structural-phenomenological modeling are of three main types:
- structural, as are all the classical categories of the theory of categories;
- phenomenological, which is a new type of category;

- structural-phenomenological, with objects formed both of structural and phenomenological parts, which is also a new type of category.

Although such categories may be considered at a very abstract level, the practice of categories and functors, used mostly in the mathematical domain, has shown, as observed before, that the best and fruitful results may be obtained for particular domains of mathematics (for Abelian groups, topological spaces etc.). When the theory of categories is used for physical theories and especially for the structural-phenomenological realms of reality, it has to be adapted to these.

3. THE OBJECTS AND MORPHISMS OF A PHENOMENOLOGICAL CATEGORY

A phenomenological category **Cphen** is a collection of phenomenological objects **s**1, **s**2, **s**3, ... where each **s** is an *elementary physical and informational object* (phenomenological sense [4] [9]) or a *set of phenomenological senses*. As an elementary object, **s** is not a mathematical object, **s** being a symbol in the formal description of the category. But **Cphen**, even if it contains only elementary objects, is a mathematical object.

However, every **s** has a physical-informational content which may be imagined, in a way, in the frame of a structural-phenomenological orthophysical theory [9].

It may happen that a part of the objects of **Cphen** to be elementary objects, in the manner described above, but another part to be objects composed of a group of phenomenological senses which do not have morphisms among them. The composed objects of **Cphen** are then sets of phenomenological senses: **s**j = {**s**j1, **s**j2,...}. **Cphen** is a category not only because is formed of phenomenological objects, elementary or composed, but also because it has morphisms among these objects.

For instance, the collection of phenomenological senses which stays at the basis of a universe, that comprises the "active information" (David Bohm, see [6]) of a universe is a phenomenological category which might be formed of elementary and especially composed objects.

The phenomenological morphisms are transformations from **s**i to **s**k. These are non-formal processes realizable in a both physical and informational way in a phenomenological realm of reality (named informatter in the orthophysical ontological model of existence [4][9]).

For each pair (**s**i , **s**k) there is, in principle, a set of morphisms Mo (**s**i , **s**k).
Every of these

uik # Mo (**s**i , **s**k) (Note. The symbol # stands for membership of a set)

being a transformation
uik : **s**i -->**s**k.

The physical-informational content of the morphism is a natural transfor-mation from a phenomenological sense to another. It does not matter where these two phenomenological senses are located. In fact, in the phenomenological realm there is no physical space, and still if we imagine these two phenomenological senses like two separated points, the agitation of one point -because it is a process, may be a sort of vibration - produces an excitation of another point which will agitate itself, vibrate, in a more or less different way. We consider, in such a case of excitation, that these two points (phenomenological senses), as processes, are "relatively neighbors" and if the phenomenological category has only such morphisms, then the category is said to be "not too large".

Concerning *the composition map of morphisms*, the situation is identical with the case of the classical structural categories, that is the composition map of two morphisms is given as

m : Mo (**s**i , **s**k) x Mo (**s**k , **s**l) --> Mo (**s**i , **s**l)

If **s**1, **s**2, **s**3, **s**4 are phenomenological objects, the composition map of the morphisms
u : **s**1 --> **s**2 ; **v** : **s**2 --> **s**3 ; **w** : **s**3--> **s**4
is associative:
w(vu) = **(wv)u**
What is supposed for the case of the phenomenological category is that these compositions are realizable in a physical and informational way in the phe-nomenological realm.

The same is the case with the identity morphism,

(**1**s) : **s** --> **s**

(**1**s)**u** = **u**

v(**1s**) = **v**

The condition to be imposed for the morphisms of a phenomenological category is to be phenomenological realizable. Therefore, excepting the phenomenological content, all the general structure of the phenomenological category is identical with that of the classical structural category. Further, perhaps, not all the various properties of structural categories are to be found also for the phenomenological categories. The particular case of the last type of categories has to be explored in the context of their use for the structural-phenomenological modeling. It is known that there are various types of morphisms of a structural category [11]. If they are phenomenological feasible, they will be used also for phenomenological categories. For instance, the *isomorphism* between two objects A and B of a category of any type, means that
u : A --> B
is an isomorphism if there exists the morphism
v : B --> A
such that
vu = (**1**A)
and
uv = (1B)
In such a case A and B are not different. This is phenomenological feasible and the notion of isomorphism is also good for phenomenological categories. If two elementary particles (like two electrons) have the same phenomenological senses (because these senses are forming, as active information, using deep energy, such particles) are not different.

4. FUNCTORS BETWEEN STRUCTURAL AND PHENOMENOLOGICAL CATEGORIES

A functor is a map between categories [11][3]. If **C1** and **C2** are two categories, a functor F between these categories is a map

A --> FA

that associates to each object A of **C1** an object FA (written also F(A)) of **C2**; and for each morphism
u : A --> D
in **C1** associates a morphism FA -->FD in **C2**, subject to the conditions of transport of structure
F(**uv**) = F(**u**)F(**v**),
and
F(**1**A) = (**1**FA).
But, as in general between each pair of objects A and D in **C1**, there is in principle a set of morphisms Mo (A,D), the map for the association of morphisms is
F(A,D) = Mo (A,D) --> Mo (FA, FB)
The functors between structural classical categories are treated in well-known books [2][11][12]. The most important cases of functors for the structural-phenomenological modeling are the *functors between structural and phenomenological categories*.

Of course, functors between phenomenological categories may also be envisaged.
Let a structural category **Cstr** and a phenomenological category **Cphen** and among them a functor F. To the structure A in the category **Cstr** is associated the phenomenological sense

FA = **s**A

in the category **Cphen**. This may be seen in an abstract way, but also as a physical and informational process. For instance, in the case of the human mind was defined an explanation gap between the neurobiological structures and phenomena of qualia or "experience". Both are recognized, the association of structure and phenomenological sense is recognized, but without explanation [13]. Being a fact of reality, it may be said that the functor between the corresponding structural and phenomenological categories is a reality, not only a mathematical concept. How this functor is realized in detail is important, but not such important at the level of an envelope theory. The functor for the structural-phenomenological modeling represents a physical and informational process. The functor itself is such a reality. The functor is a feasible reality realizing the coupling between a structure and informatter (referred to in the previous chapter).

A series of problems are opened at this step of our reasoning. A neurobiological structure may be a *category of neuronal automata*, and in general *categories of automata are also to be considered.* An automaton may be considered as a category, of which objects are its states. Each state is a structure, a set, and the morphisms between the objects are therefore also functions (from a functional point of view, relations and functions among sets were named formal functions [10]). A category of automata is then a category of categories. Each object is a category with an automaton with many states that are the objects of this automaton. The morhisms of the category of automata are maps from an automaton to another. This may be the case of the maps among various parts of the brain. In such a case, it might possible that the association of phenomenological senses to depend, acting subject to some conditions, on more functors between a structural and a phenomenological. By analogy with the functional architecture [10], a *functorial architecture* could be defined.

Between **Cstr** and **Cphen** the functor F associates also morphisms of the first with morphisms of the second. If **u** is a morphism in **Cstr**, then F**u** is a morphism in **Cphen**. This is feasible from a physical and informational point of view, because to a transformation of structure, a transformation of phenomenological sense is necessary.

The above-described functor may be called a *structural-phenomenological functor*. Because also the reverse process is feasible, a *phenomenological-structural functor* R may be defined between **Cphen** and **Cstr**. For this case, to a phenomenological sense corresponds a structure, and to a phenomenological morphism a structural morphism. Therefore, F and R have to work together, the first one enhancing qualia and experience, the second one bringing intuition and creation in the functioning of a mind. These functors are present in any organism under forms that will be explored in future papers. Two categories and two functors mainly de-scribe an organism

Org = <**Cstr**, **Cphen**, F, R> ,

and of course other items will be necessary to be mentioned in this enumeration for various types of organisms.

The functorial architecture between a structural category and a phenomenological category has always two such functors F and R, which are of fundamental importance in the real world and in its structural-phenomenological description,

5. STRUCTURAL-PHENOMENOLOGICAL CATEGORIES

The objects of a structural-phenomenological category are pairs of structural and phenomenological objects (A, **s**), if A and **s** correspond to each other by a *functorial link*. At the origin of a structural-phenomenological category there are, in such a case, two categories **Cstr** and **Cphen** among which there are functorial links.

If **Cstr** has the objects A, B, C, ..., the corresponding **Cphen** has the objects FA, FB, FC,... which are phenomenological senses, and F is the functor from **Cstr** to **Cphen**.

The resulting structural-phenomenological category is not the product **Cstr** x **Cphen** of the above two categories, but only a subcategory **C'str-phen** of this product. Indeed, the product [11] of the mentioned categories (retaining, for clarity, only three objects) contain the objects

(A, FA), (B, FB), (C, FC); (A, FB), (A, FC), (B, FA), (B, FC), (C,FA), (C,FB),

but only the first three pairs are feasible in the reality of the organisms (Every structure has its phenomenological sense). The subcategory **C'str-phen** contains a part of the objects of the product category **Cstr** x **Cphen** , contains also all the morhisms of **Cstr** x **Cphen** among the three pairs of objects mentioned above, maintains the composition of the respective morphisms, and the identity morphisms. More, because **C'str-phen** contains all the morphisms of **Cstr** x **Cphen** among the three pairs of objects, it is a *full subcategory* [11].

It may be observed that although the product **Cstr** x **Cphen** formally seems to be a structural-phenomenological category, this is not true because the condition of feasibility in the real world is not fulfilled. The full subcategory **C'str-phen** is then a structural-phenomenological category.

The *feasibility condition* is essential for the adaptation of the theory of categories to a mathematical structural-phenomenological theory of categories.

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Communication at the Section for the Science and Technology of Information of Romanian Academy, September 18, 2000.