INFORMATIC VERSUS PHYSICAL ONTOLOGY. ONTOLOGICAL FUNDAMENTALS OF MATHEMATICS

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Abstract

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Summary: Mathematics and the ontological model of the Ring on Existence (R.E.); Informatic ontology and physical ontology; Formal and Intuitive; Set as an ontological-objective idea; Ontological implications of paradoxes; The consequences of the paradox “Set of all sets”; Formal universe, mental unity, ontological unity.

MATHEMATICS AND THE ONTOLOGICAL MODEL OF THE RING OF EXISTENCE (R.E.)

The ontological model of the Ring of Existence (R.E.) may have serious implication on physics and biology and also on the science of information.

Knowing what is the role of mathematics in actual science, one may obviously ask the following question: to what extend could the influence of the new ontological cognitive framework get upon mathematics, as the specific field of science and also upon the role of mathematics in the description of reality ?

We shall deal with such problems in this paper. Some observations on the position of mathematics related to the ontological model R.E., presented in a previous volume [1], were, essentially, the following:

a) Because all mathematics could be expressed by the theory of sets, the occurrence of paradoxes within theory "reflects" the peculiarities of human knowledge [2] and also the fact that the notion of "set" cannot be roughly separated from the nature of human being abstracting mental process; that is why "the paradoxes of the theory of sets may form the subject of a real philosophical experiment implying the entire reality of the human being"[3].

b) The mental reality of the human being is not understood only by its space-temporal constitution but also by the intro- opening in the informatter belonging to the orthoexistence and as “no concept can exist without any support" [4], any abstract concept is generated within an ontological context (space-temporal and informaterial).

Operating within an ontological context but first of all mentally-psihologically, this concept may be looked upon as an abstract reality. This is also the way to understand the mathematical object [5].

The mathematical object is a concept, an idea. As an abstract reality it operates psychologically but, in fact its basis is ontological. May this "ontological" have any kind of influence on the concepts ? Do the paradoxes signal out indirectly on this, that is the ontological basis of the concepts ?

c) The fact that the axiomatic theory of sets avoids the paradoxes does not mean avoiding of the ontological too. Although an axiomatic system avoids the paradoxes, their track still remains, in a way, due to the inconsistency of the system. The inconsistency means that a formal, logical, logical- mathematical system cannot close in itself. As Gödel demonstrated this though his theorem on incompletitude, the result is that the opening, the incompletitude is a fundamental law of a formal logical-mathematical system [6]. In this case, is the opening of a formal system an opening towards the ontological ? It is true that a formal system, inevitably open, may be closed with another mathematical reality [7], but the latter, as a formal system, may have an opening and so an, until reaching an ontological reality where this process stops [8].

In the fact that a formal logical-mathematical system does not logically close is a legity, then the problem of consistency is wrongly approached: "the normal condition of the mathematical systems is to exist without logical closing and, from our point of view they are not obligatory inconsistent" [9].

The new ontological model does not raise only problems such as: **a** (mathematics implies a philosophical experiment in its fundamental aspects), **b** (the nature of mathematical objects) and **c** (the logical opening as the normal condition of a formal system) but it also implies reaproaching the problem of space primordiality or of the set primordiality within the mathematical concepts, that of the theory of sets status (**d**) and also of the ratio between the formal and non-formal in the description of reality (**e**).

INFORMATIC ONTOLOGY AND THE PHYSICAL ONTOLOGY

It is known that the theory of sets paradoxes gave birth to a variety of philosophical attitudes: the intuitionism (named sometimes mathematical antropologism too), the classical platonism, the logicism and the formalism.

Among all these only the platonism is ontological but it represents the case of an ontologism related to the existing mathematical objects as platonicist ideas. For platonicist mathematicians, the objects, the mathematical structures exist independently of the mathematicians' thought. It removes the possibility of any creation in mathematics, the mathematical truth may be only discovered.

What remains positive from platonism is the affirmation of the objectivity of mathematical objects. But platonism is not reduced only to this; according to it, the ideas exist beyond the

matter (objective idealism). In this last sense platonism is absolute but sometimes some of the mathematicians understand any separation of the abstract objects from the subject as platonism.

The most obvious example is presented by Paul Bernays, the author of the axiomatic theory of sets [10], developed between 1937 - 1954.

For Bernays, platonism is dominant in mathematics and the logicism and the formalism are just forms of platonism.

Only the intuitionism is opposed to platonism but, according to Bernays [11], the former is complementary to the latter.

Bernays rejects the *absolute platonism* for the reason that due to it the paradoxes of the theory of sets in its initial form [12] were produced. He is the adept of a *restrictive platonism* which admits the objectivity of the mathematical objects "as an ideal projection of the thinking area" [13]. This is also valid both for logics and for mathematics. The starting points of the logical and mathematical theories are, according to Bernays, platonistic suppositions in the sense of restrictive platonism and that is why the logicism and formalism could belong to platonism from the philosophical point of view.

As the platonism framework is restricted, one may consider that the paradoxes of the theory of sets could be avoided as the axioms avoiding them are just platonistic suppositions; Bernays considers that the restrictive platonism represents a philosophy which does not generate paradoxes.

Bernays' restrictive platonism recognizes an objective existence of the mathematical structures which is neither ideal nor material.

Is this possible ? Neither subjective representation, nor a material existence and nor an ideal existence ? This was possible for Frege too as he had built a *formal ontology* [14] which is different from the *material ontology* or from the *ideal ontology* (platonically absolute). With Frege, the formal ontology is a restrictive platonism because he rejects the identification of a mathematical or logical object with a symbol having an arbitrarily imposed content. For Frege logics and mathematics (which he hoped to be reducible to logics) are just theories for same *objective entities* (forenamed objects containing, in general, both the concepts and the objects of Frege's theory [15]), *existing thus independently of consciousness*.

Having in view the fact that actually any mathematical or logical theory may function as an informational structure in a computer, it is obvious that the logical and mathematical structures are provided, in a way, with an artificial life, the result being that, with Frege, the formal ontology is conceived as an ontology of information which might be named rather *informatic ontology*. We prefer to adopt the word "informatic" because the logical and mathematical structures may operate not only in human beings consciousness but particularly in the computing intelligence and also in the artificial intelligence of computers.

The notion of "informational ontology" is deeper as compared to the one of "informatic ontology" because it also is related to the intimate processes of the existence until reaching the depths. The informational ontology also refers to the mental processes. The informatic ontology belongs to the existence of those structures which generate and are introduced from the consciousness in the artificial intelligence.

These structures have no ideal prototypes and they are not even material. The formal logical-mathematical structures are informatic structures. They are neither ideal nor material, they are merely logical-mathematical or informatic structures. The word "restricted platonism" is not adequately chosen here, although it reflects, to a certain extend, the informatic reality of the formal, logical-mathematical structures. These structures are not absolutely ideal due to the fact that they function within the artificial intelligence.

Only the natural intelligence and the natural consciousness have ideas.

In other words, the restricted platonism, the formal ontology and the informatic ontology seem to be equivalent notions.

The formal informational structures are not ideal in the absolutely platonic sense, they are not even matter but only a form in the matter because they may be located in one or another material substratum.

The formal informational structures which we shall name "informatic structures" may enjoy the related significance properties and the contextual ones when they have the possibility of a dynamic self development as in case of a data base (or knowledge base) and as in case of an informatic program within a computer.

Being not matter, the informatic structures are ideal, but in a restrictive sense. They might be named ideal structures with a material substratum and as the formal informational structures are essential while the material support may be a common one (neuronal, made of silicon, of magnetic substances, etc.) their restricted ideal character is more important than their material support. Having in view the ever growing importance of the notion of "information" in contemporary science, we should rather name such semi-ideal structures *informatic structures*.

Not all informational structures are also "informatic". The mental processes are informational processes having an informatic component without being reduced only to that component. They also include processes of phenomenological sense [16]. That is why the complete informational structures , in their philosophical acception, are ones having both a formal component and a phenomenological sense component.

So, we may distinguish there types of information:

- the formal information (= informatic structure);

- the phenomenological information;

- the complete information (formal-phenomenological).

A mathematical theory may develop as a formal information, that is as an informatic structure, both when written on paper and when processed in a computer.

From this point of view the ontology of mathematics seems to be like an informatic ontology.

In human mind the "informatic" is complemented by the "phenomenological", making thus mathematics exceed in fact the informatic ontology (= the formal ontology and the restricted platonism). Further on, one may ask the following question: to what extend may the informatic ontology be independent of a real ontology ? The only rational answer is that the informatic existence is derived from the material existence.

As we have seen before, the informatic ontology is just part of a larger informational ontology and the informational ontology is only a part of an ontology of existence which refers to the material, informational and social aspects. There is no way of offering an independent status to the informational ontology. In fact the problem is the following: if the informational is as primary as the matter, it is independent of the matter. As the matter also presents informational aspects from its roots, too, it appears at the same time with the material, and being equal with the material it could never exceed it; it could never exist without the material.

To sum up, the logicism and the formalism are based on a specific ontology, an informatic, formal ontology. It may be looked upon separately from a material ontology only for the separate study of the logic, mathematics and informatics.

Only in such circumstances Rudolf Carnap's affirmation may be explicable: "the logic treats only about possible entities and may not affirm that something exists or does not exist" [17]. In reality only the link between the informatic ontology and the material ontology assures the objectivity of the logical and mathematical structures. The two ontologies form just one, "the ontology of the existence" the former being part of the latter.

FORMAL AND INTUITIVE

Until the second half of our century the phenomenon of functioning of certain formal informational structures, that have become more and more intelligent and located outside the human mind, was not known.

The logical and mathematical structures in the informational form, as informatic programs and data structures and ultimately as symbols operate and may determine actions in the material reality which are independent of man and society (even if they were introduced by the society). Such structures, without being material exist, operate. Nobody may doubt their ontology. We have also noticed that they copy nothing, neither the platonicist objective ideas nor the matter. They generate from matter in an unavoidable way.

Certain deeper principles should then exists; they are meant to explain why such structures are born. It is obvious that the occurrence of such structures is manifested, first if all, on a mental-psychological plane, within a social context for the human observer.

What is then the status of psychology and particularly of the intuitionism related to the informatic and logical-mathematical structures ?

It is also obvious Frege's remark that our thought, although subjective, has an objective content. This fact is demonstrated writing the logical-mathematical structures on the paper but particularly in an informatic form.

In the second case they prove to be experimentally objective. Informatics has achieved, in practice, one of Frege's principles, namely: (1) we must always make a clear-cut, distinguish the "psychological" from the "logical", the "subjective" from the "objective" [18]. Moreover, the informatics and the artificial intelligence satisfy Frege's second principle, too: (2) the words significance should not be searched in their isolation but only in the context of the sentence [19]. If we do not apply principle (2) then Frege says: "we are obliged to accept mental images or actions of the individual mind [20] as words significance, being thus in contradiction with principle (1).

Due to this, Frege restricts the semantic investigations to the *logical semantics*, the founder of which he is.

The logical semantics and the semantics of the artificial intelligence are 1st class semantics while the semantics implying the mental-psychological aspect is a superior one belonging to the 2nd class. Having in view the two kinds of semantics the problem is if the formal structure consumes the whole content of a mathematical object. May the mathematical object be reduced only to its formal structure ? If not, what does the mental-psychological aspect brings along from the point of view of what is non-formal and may be objective at the same time ?

From the point of view of mathematics, the mental-psychological aspect brings along, first of all, the *intuition*,.What is the objective content of the intuition ? The new ontological cognitive model to which we refer in this study leads to the fact that the intuition is interpreted as being a "bridge" between the mental-psychological aspect and the immediate reality, both the space temporal one and the deep, informaterial one. Thus the intuition is, from our point of view not only mental-psychological but also ontological.

In case of mathematical intuitionism, L.E.J. Brower eliminates the material or ideal ontologism which may influence the intuition [22] and, on the other hand, he does not admit that mathematics may develop independently of the human mind: "The problem where the mathematical accuracy exists is seen differently...; the intuitionist says: in the human intellect; the formalist says: just on the paper" [23]. Because mathematics is a product of the human mind the intuitionism does not admit the existence of mathematical objects independent of human thought: "their existence is guaranteed only to the extend they may be determined by thought" [24].

We should notice that the rejection of any ontologism, not admitting even the existence of the restricted platonism (the informatic ontologism) which, as we have seen, must be related to a deeper, material ontology, leads to subjectivism. And, if the intuition is just subjective then how could it discover objective truths ? In reality, the intuitionism should be also interpreted from the ontological point of view. In this respect it cannot be rejected. This is also admitted by M. Bourbaki who, when formally treating on mathematics, does not give up the intuition, about which he says: "the mathematician's intuition which is not necessarily of space and sensitive nature, as we believe sometimes, consists rather of a certain knowledge on mathematical beings (objects) very often assisted by images of various nature" [25].

The blending between the intuitive and the formal represents the way in which mathematics exists in human mind. The artificial intelligence can only include the formal aspect of mathematics. Sometimes the mathematical treating emphasizes the intuition (the intuitive or naive theory of sets containing also much of the formal aspect), some other times the formal, but, in fact, the two aspects are almost never separated.

If a completely formal treating of mathematics is possible, starting with the fundamentals and going on up to its last consequences, then mathematics could be given in charge of the artificial intelligence. It is true that very many intuitive aspects may be formally treated but this is valid until reaching the fundamentals of mathematics. Here the formal cannot close and we have only to approach *the intuition which proves to be effectively complementary to the formal.* In the development of a certain area of mathematics the intuition may replace the formal and, to this formal an intuition may be found. They may replace one another but it is better, in fact, to go on together. This is also observed by P. Bernays [26], whose role in the formal theory of sets is well-known. Consequently, *a mathematical object is more than a formal structure*[27]. That is why the artificial intelligence can receive only the formal structure and not the mathematical object itself (once more - by object we mean the conceptual object). In this respect the intuitionists are also right when they do not admit an existence which is independent of the human thinking for mathematical objects but they are not right when they refer to the formal structure of these objects.

The formal structure is potentially informatic and may by implemented on a computer or in an artificial intelligence program.

If we watch a vivid mathematical speech carefully, the intuitive aspects, due to the images they create, seem to lead, equivalently, on the mental-psychological plane, to concrete objects, the intuitive seem to leads to a certain reality, even if the implied notions are abstract. The above-mentioned affirmation has an experimental value which anyone may check, observing, for instance, a formal and abstract text like the one of the theory of sets written by M. Bourbaki. Many times the intuitive seems "to short-circuit" the formal, arid demonstrations and that formal demonstration, necessary for rigurousity, seems to respect the intuitive. If it doesn't respect it, the intuition was not clear enough; it should be corrected; it is necessary an agreement between the intuitive and the formal. Very many relations are intuitively observed and then rigorously and formally demonstrated. The intuition inevitably occurs in the operation of the human mind, even when the aim is watching of mathematical, formalized demonstrations. That is why, working

only formally means the introduction of the human being in a mental machine and informatizing him. Such a requirement is considered non-realistic and even delirant by Rene Thom [28].

The intuition always goes along with the normal operation of the human mind.

It is true, as Frege says, that "a mathematician can perform complicated calculations without understanding something intuitive by respective signs, something perceivable through senses" [29]. In spite of these facts the human mind will draw an intuition on these thing formally met, including a certain emotional component. The human mind cannot operate non- intuitively without imposing to itself to work formally for a while. That is why the removing of the mental-psychological aspect by Frege, and through it the removing of the intuition too, has the importance of a separation method *up to the limit the formal and the intuitive may be separated*. This separation provides the formal with that kind of objectivity which we saw that it was experimentally checked up using the informatic method.

What about the intuition ? Does it bring along something objective too ? When we intuit something which could be formally transcribed, provided the formal is objective, was the respective intuition objective too ? If a certain intuition may be objective it means that, in general, we may also find objective intuitions among the various ones. To be objective means not to be subjective, not reduce anything to the subject, *it means that the intuition brings along something belonging to the nature of things and that it has also an ontological character.*

Our conclusion is that the intuition is not only mental- psychological but also ontological.

That is why the mathematical intuitionism which makes just a half step to the ontologism has not succeeded in being convincing. If the intuition (and not the intuitionism) is an ontological reflection then the mathematical objects are an ontological reflection, too. *Thus mathematics appears then, on the whole, as an ontological reflection.*

What about the mathematical creation ? Could mathematical objects be created without having an ontological source ? A new mathematical object may be invented due to intuition within a creative process. Mathematical objects may be discovered through intuition. Any mathematical object should also have an ontological correspondent reflecting ontological realities or performing ontological potentialities. That is why the philosophical and the existential role of mathematics grows enormously within this vision determined by the ontological model of the R.E.

SET AS ONTOLOGICAL-OBJECTIVE IDEA

The intuition operates neither with words nor with ideas within the human mind.

The idea, based on an intuition is the successor of that intuition. Not even a new mental image, as it is, is not merely an intuition but it derives from an intuition or awakens an intuition. What is so closely linked to the intuition is the occurrence of a phenomenological sense which only later

on will have the outline of a mental image or of an idea. The images and the ideas are intuited when they awaken a sense. This sense belongs to what we named before the high class semantics (the 2nd class semantics).

The sense is a phenomenon produced due to the access of the living organism to the informatter. The informatter has naturally, in itself, the orthosense of "to be"; that is why any living creature may experiment the sense of to be, it may have that sense that we call "beingness" as in the terminology used in "The Depths of Existence". This sense is obviously an ontological one. "Beingness" [30] determines a certain space-temporal structure in the human mind. Attaching of this sense, in this case "to be", to the respective structure determined in mind is an intuition. The intuition is a mental-psychological process. Not all the living organisms have psychological processes; only those which can attach to the sense of beingness a structure in the nervous system. The assembly between the structure and its sense represents the idea, in this case the idea of "to be". *We intuit ideas when we attach a structure to a sense or a sense to a structure.* The idea of "to be" is somehow spontaneously intuited, it represents the assembly between the structure and the sense, as they occured naturally, not created, not searched, being in fact ontological. The structure of "to be" belonging to the idea of "to be" is not, at this very moment, a verb, a linguistic structure, it is not an image either but it is a certain neuronal structure naturally attached to the sense of "to be". And the sense is a physical process in the informatter, a different type of structure from all the types we are accustomed to, which we name phenomenological process.

The word "to be" is derived from the idea of "to be" and ultimately it is linked to the sense of "to be" but, as we have also noticed, "recording" of the "beingness" through a condition (state), a large "symbol" branched in the nervous system... (the symbol) will become more ordered but maybe more limited, expressed by the word "to be" [31].

The word "to be" cannot express all the richness of the sense of "to be" but it will render what it is essential for the human being in his practical life. The natural language may restrict more or less of the primary sense, according to the realities man faces in his practical life but nothing could stop a word to represent or to bring back all the original sense. It is not the natural language we suggest to speak about now. We shall see the way in which the word "to be", in the context of the natural, concrete language receives the formal significance of "to be" which is exactly in the sense Frege wished it, that is within a 1st class semantics, within a formal semantics respectively. The way from the sense to the structure may also be traveled through, vice versa, from the structure to the sense.

The human mind, being at the same time in the posession of the structure "to be" and of the sense "to be", is implied in the *recording* of this double condition (which is an idea in its unity) and, thus, it will generate a *new* structure to which it will attach a sense in the informatter. Recording of the dual condition in the form of a new structure and searching the sense of this new structure must be again an objective process.

Recording of the above-mentioned dual condition is in fact the most elementary and fundamental act of knowledge and that is why the sense which is attached to the new structure should be the one of "to know" (the sense of "to know" and not the respective word). The informatter owns the

sense of "to be", it may generate other senses too, but it also admits new senses, induced by space-temporal structures. Referring to the human mind - it admits the sense of "to know". This is the way to explain the occurrence of the mental-psychological phenomenon of consciousness as the unity of the ideas "to be" and "to know".

The mental-psychological aspect appears objectively. The subjective and creation liberty is due to its nature, to its power of delivering itself from the objectivity of the link between structure and senses. That is why, the mental- psychological aspect may be a blend between the objective and the subjective; within it the imagination, the creation and the will may develop.

Some of the ideas existing in the mental-psychological bearing are ontologically objective, in the sense that they occur objectively, as for instance the ideas of "to be" or "to know"; the same for the logic. Other ideas, subjective, imagined or created, may be possible ontological. They are not objectively ontological as they are not directly derived from or discovered in the existing ontological aspect. Although, even if they are imagined or created they imply a structure (neuronal processes) and a sense (informaterial processes) and thus, they have an ontological content. The idea, being a mental-psychological reality, we shall say that any idea is ontological when it is ontologically objective, that is directly derived and discovered from the existing ontological aspect. The other ideas, subjective, imagined, created in this restrictive sense are not ontological.

Having in view the above-mentioned facts, the set being a concept, an idea, has a sense (an informaterial one) and this sense must be ontologically-objectively created.

The set is not a creation which we impose to the mankind but an objective reality with ontological roots. The intuition of set is unavoidable in the operation of the human mind. The set is obtained through an ontologically-objective intuition. The observation of physical objects sets is not the direct ontological root of the abstract notion of the set. The sets of physical objects may stimulate the generation of the set notion coming from deeper ontological roots. ***The set should be a notion linked of certain intrinsic properties of the informatter.*** The set is a far more primordial notion that such implications do not occur (due to the ontological model we adopted). Thus *the set cannot be a purely formal structure* it has also an intuitive content and if we do not remain suspended in the intuition, fact we proved not to be possible, *it has also a deep, ontological content*. Could we distinguish what this content is ? The paradoxes of the theory of sets are just the signal for the deep ontological character of the set.

ONTOLOGICAL IMPLICATIONS OF PARADOXES

It is known that even Cantor himself, the creator of the theory of sets, in his elaborated after 1878, discovers, in 1895, an antinomy in this theory [32]. The paradox of the theory of sets or of the type of the theory of sets (as it is the Burali-Forti paradox, 1897, found, in fact, by Cantor too in 1895) was going to become Cantor's preoccupation, as he sometimes expressed his opinion on it, like in 1899, in his letters to Dedekind.

Cantor says that one cannot speak about the set of all sets without entering a contradiction. Essentially this is the paradox of the theory of sets and it is of logical nature. R. Carnap and

others showed that the paradoxes appearing in the theory of sets are logical paradoxes and B. Russel emphasized as clearly as possible that they are common both for logics and for mathematics. Gödel, also, considers that Russel's most important investigation is the one in the field of analyzing the concepts of the formal logics, namely concerning the logical paradoxes and their solutions.

Gödel says about Russel the following: "Analyzing the paradoxes to which Cantor's theory of sets has led, he delivered them of any mathematical technicalities, revealing the amazing fact that our logical intuitions (that is the intuitions regarding such notions as: the truth , the concept, the existence, the class, etc.) are self-contradictory" [33].

The fundamental paradox of the theory of sets was named *Russel's paradox* although the name of Cantor-Russel's paradox would be more adequate. We are not going to deal now with the form in which it was expressed by Russel but with its simplest form, "the set of all sets" (SAS) as here, a philosophical experiment (problem **a** from the beginning of this paper) is the most obviously hidden. This is why we shall also leave, for the moment, aside the various solutions for the paradoxes through the theory of types (Russel) or through axiomatization of the theory of sets (the Zermelo-Fraenkel system or the von Neumann-Gödel- Bernays system).

In other words, we shall start from the very beginning having in view the new ontological model. When we say "the set of all sets" (SAS), the classical, logical interpretation leads either to the conclusion that the SAS does not exist because of the paradox this formulation generates, or that the SAS exists, but it is anything else and not a set. The "rigorous" classical logics admits neither the former nor the latter version, as Frege himself showed. After he received Russel's letter in 1902, in which he informs about the paradox found in the arithmetics Fregeean theory, Frege wrote: "Even now I cannot see how arithmetics may be scientifically fundamented, how the numbers may be thought as logical objects submitted to the study, if are not permitted, on condition, at least, to pass from a concept to its extension" [34].

Frege cannot admit, within a rigorous classical logics, that a concept as it is the concept of set, could not be extended to the SAS too. Although this is not possible because the classical logics becomes contradictory to itself. The SAS, either does not exist, or it is anything else and not a set, defying thus the classical logic.

Octav Onicescu [35] considers that the SAS is not a mathematical object because a mathematical object may be not a contradictory one.

The contradiction is not denied but it is considered a proof of non-existence for the mathematical object: "Essentially mathematics do not deny the op.cit logical antinomies, but, based on those contradictions they reveal, they deny the quality of mathematical objects [36] for those pseudo-objects to which these antinomies are related".

In another essay, Octav Onicescu made the following remark concerning which he considers to be the most important of the paradoxes - the concept represented by the set of all sets: "Its discussion consumed much ink and much useless ingeniosity. The problem which must be solved first to answer if the affirmation is or not paradoxal from the mathematical point of view is if it

belongs to mathematics, if it corresponds to a mathematical object having all required characteristics. It is not enough to be formulated in terms of mathematics for a concept to belong to this science" [37].

Provided the SAS is not a mathematical object, what else could it be ? Why does Octav Onicescu suggest that it could be an object of a different nature ? What is the nature of this object ? This is, obviously a philosophical question.

We shall remark how this paradox sents us to another reality which, according to the ontological model of the R. E. cannot be otherwise but deep. Could the SAS, possible, be just an ontological reality ?

In fact the SAS develop on the mental plane but at a formal, informatic level, we may say (see the next paragraph) in fact, neuronal. The sense delivered in mind by the SAS related to the formal, logical thinking leads to a contradiction, in fact to a lack of sense as compared to the sense of the set. Even the respective lack of sense (nonsense) is in fact a sense and the question is: what is the objective content, although hidden, of the new sense ?

If, ontologically, the set operates at the formal, neuronal and informatic level, the SAS, on the contrary, should correspond to another level. For sure, the SAS offers the human mind one of the most important philosophical experiments.

Hereby it is useful to remind another philosophical experiment in which logic is implied again: it is the one of existence and non-existence (ENE).

The fact that the relation existence-universe appears on the logical plane, both as existence and as non-existence (see the entire history of philosophy), has the value of a philosophical experiment leading to the separation of a deeper ontological reality, the orthoexistence [38]. This is the way to explain the ENE contradiction. The ENE contradiction appears as objective at the universe bearing. As soon as we recognize a deeper existence than the universe, it - here we mean the existence and together with it the ENE contradiction - becomes explainable.

If we consider the ENE just a logical contradiction without taking into account the fact that it reflects a deeper reality, it is obvious that we have no solution and the ENE appears as an absurd affirmation. The ENE is not, in fact, an ontological reality, but an affirmation on it if we take into account the deepness of reality. There cannot be any doubt on this within the model of the R. E. Is it possible in such a case that the SAS should similarly indicate a deeper reality, maybe the same deep reality as the ENE ?

We could bring along another analogue example and, of course, there may be others too. What we speak about is the corpuscular and ondulatory aspect of an elementary particle. The two effects are, indeed, complementary, as Bohr says. Bohr's complementarity is an affirmation which takes into account the more intimate, deeper nature of the elementary particle.

The corpuscle and the wave is a combination which is a paradox from the logical point of view (C W) but it must be accepted because it reflects a reality.

The paradoxes CW, ENE and SAS are revealing truths. Analyzing the phenomena of the quantum mechanics, of the elementary particles and of the quantum theory of field, Fritjof Capra also comes to the conclusion that there is a deeper reality, to the conception of the depths unit having a corespondent only in the Hindu, Budist (in all its versions) philosophies and in the Chinese, Taoist thinking. We are not going to underline here the fact that, according to Fr. Capra, conceiving of the deep unity is done only mystically but we only notice that one of these philosophies, namely the Taoist philosophy may be also interpreted as a materialistic philosophy. What is important is the fact that Fritjof Capra notices that "both the physicist and the mystic (meaning, in the correct acception, any Oriental philosopher of the Hindu, Budist, Tao, Zen type, etc.)wish to communicate on their knowledge but when they do it using the words, their enunciations are paradoxal and full of logical contradictions" [40]. Neither the language, nor the imagination can cover very well the corpuscule-wave (CW) reality, says F. Capra. He also affirms: "Any time the essential nature of things is analyzed by the intelect it seems absurd or paradoxal" [41]. The analysis of CW complementarity shows that while the corpuscle aspect is concrete the wave aspect is abstract. Indeed, the quantum mechanics attaches probability waves to the particles. The probability waves are not physical ones but they are waves reflecting a certain behaviour of particles.

Only in the case of the electromagnetical waves, of light, the CW aspect appears as it is: photon-wave. In the most general case, "the introduction of the probability waves, in a way, solves the paradox of the particles as being waves, putting them in a new context; meanwhile we are led to another pair of opposed concepts which is still more fundamental, the one of existence and non-existence... We can never say that an atomic particle exists in a certain place, but we cannot also say that it does not exist... so (it) manifest a strange kind of physical reality between existence and non-existence" [42]. It is obvious that the existence and non-existence of the particle appears at the universe bearing as within the universe spare the particle is not to be found in a certain plan, but it is not absent too.

We saw that the philosophical experiment of existence urges us to go to a deeper bearing of the orthoexistence. Physics requires the same: in case of the atomic physics we have to go beyond the existence and non-existence concepts [43]. Again, the concepts oblige us to admit objectively when they become paradoxal, the existence of a deeper reality. This appears abvious from the philosophical point of view, in general, and not only due to the oriental misticism as Fr. Capra thinks.

What is important is the fact that the SAS, ENE and CW are placed in similar logical situations and it is obvious that in the cases of ENE and CW they lead to a deeper ontological reality. Why not the same for the "purely" logical case of the SAS ? If we admit that logic appeared to reflect the reality (leaving aside the possibility of creating a logic to build a reality) we have to expect that the SAS leads us to a ontological reality too.

A link between the SAS and the CW is to be found even with a logician like Quine [44].

He finds an analogy not only between the SAS and the CW but also between Gödel's demonstration on the occurence of indecidable enunciations in arithmetics (in 1931) and Heisenberg's non-determination principle in physics [45]. The approaching, from the

philosophical point of view between the ENE, CW and the SAS, mentioned in a previous paper [46] led to the enunciation of problem **a** in this study.

THE CONSEQUENCES OF THE PARADOX OF THE SET OF ALL SETS (SAS)

Let us come back to the SAS paradoxe. Acording to that we have above, we intuit that the SAS leads us to a deep ontological reality. We shall note this reality by *1* because we suppose it is the informatter as a unity. Therefore

SAS ---> *1* (1)

the sign ---> showing not an implication but an intuition.

How could we interpet expression (1) ?

Let us come back to the expression "the set of all sets", to perform, mentally, for the moment, the operation "the set of all sets ". This implies to form during a first moment a set from the existing; in the following moment we have to take into account the new set and to form again the set of all sets and so on.

The SAS expression is not on expression reflecting a static logic but rather a dynamic process (having a "dynamic" logic).

When we affirm "a set of all sets", the operation is perfectly possible within a classical, static logic.

On the contrary, "the set of all sets" is a dynamic expression if not temporal; it has anyway a measure or a rhythm (corresponding to the clock of a computer or to an ontological rhythm [47]).

The SAS expression looked upon within the classical (and static) logic seems to be paradoxal and in such a case we are not going to insist on this. If we look upon it as a dynamic expression then the SAS has got a sense , a development in rhythm or time of the SAS search. To what extent? Until t = infinite? Or may be for t tends towards the infinite?

Now we are going to fall into an important digression. Let us consider the expression "the sum of all sums " (SOAS) which is analogue to the SAS expression but whih could be operated on a computer. It is obvious that if we locate the SOAS expression within the static logic, it becomes paradoxal. In spite of this fact the computer admits it. If the computer has the SOAS instruction, it will chose a series of S terms, will sum them up then it will wonder if there is still another S term, it will find that there is (it is just that sum calculated before), it will add this sum to the

previous one, it will get a new sum, it will wonder again if there is on S term, it will find that there is, etc. The algorithm is easy to be established. The SOAS instruction gives birth to a repetitive cycle and the computer will calculate the SOAS as long as the energy and memory is available for it to operate.

There are still certain necessary observations. The SOAS expression is not absurd ; it is operational, it operates in time (or rhythm) even if this does not lead to a final result, because the computer does not naturally stop. The SOAS is obeyed to a dynamic logic which, in case of the SOAS results in the indefinite growing of the SOAS. The fact that which expression was introduced in the computer reflects the objective character of the dynamic logic . And the fact that the time, the rhythm interfases in the SOAS expression reveals the ontologycal character of logic. The time does not interfere only in our mind , according to Brower's affirmation , who retains only the time apriori from the space and time apriori of Kant, but also in the computer *for the same logical expression* . On the contrary, Brower is right to find up that the SAS (or the SOAS in this case) is not submitted to the principle of the excluded third because we may say anything else about the SAS (or SOAS) except affirming that is exists or does not exist.

As for the SAS, besides saying that it exists or it does not exist (in the static sense of logic) one may also say that it *exists also differently* and it exists, in fact, only as "*exists also differently"*.

Thus the dynamic logic leads us on a new coordinate, of time or rhythm, towards a "third" which we cannot exclude. As within this "third" the time and the rhythm interfere, so the interference of the ontological aspect is obvious. The SAS expression becomes indeed concretely enough ontological.

It can operate only in a reality having rhithm or time . We come to the conclusion that the dotted arrow from expression (1) begins indeed to stand out ontologically too.

What are we going to do with such logical objects as the SOAS or SAS ? They are so dynamic that it is obvious, the SOAS would never stop if we had infinite source of energy and infinite memory. A logical object like the SOAS does not simply only the time but also the energy. Its tendency would be to consume all the energy of the universe had not an infinite energy. The existence of the SOAS leads us to ontological implications which are by far more powerful than the time, implicating also the energy of the universe.

What about the SAS ? It develops on the mental plane because the SAS is a concept. Statically looked upon, the SAS expression is paradoxal and we deny its existence although we come in contradiction with the common logic (the static logic). Dynamically looked upon, the SAS exists but is an object which we grasp but which always escape, undecidable whether we grasp it or not, if it exists or if it doesn't exist in any finite time . We must come look indeed to the question: what happens at the infinite ? What are we going to decide when reaching it ?

The fact that the SAS exists in the dynamic sense and although in any moment or rhythm of the dynamics, statically seen it is undecidable if it exists or not - all these lead us to the idea that in fact the SAS exists and, as any well-determined concept it must be a *static concept* too.

All dynamicity of the SAS expression and concept must the reflected in a new static concept, towards which the dynamic SAS tends . Let it be this, *1*.

Thus we must not expect t = infinite but even only for t ends towards infinite we may assgin the SAS the significance of the concept *1*. At the limit the SAS tends towards *1*, as the SAS implies the *1*. But what does the *1* represent if *it is not* the set of all sets and still it *is the consequence* of the set of all sets, this time in a static acceptance ? It cannot be anything else but the general *framework in which both the statics and the dynamics of all sets develops*. The significance of *1* is, according to this, *the framework in which all the sets develop*. Being a concept, *1* should also have a sense. As an idea, at least, it should have a sense.The *1* represents all that could include everthing, it represents the informatter for concepts or the orthoexistence for the concrete universe; it may also represent the unit of the whole cosmos.

It is only in *1* that statics and dynamics could join. *All the sets are born in fact from *1* and they do not build the *1*.* Everthing, concepts and reality start from a unity and they develop *from* and in this unity. From this unity, according to the properties of the informatter, the sets are generated, from which, some of them coupled with the energomatter become an universe, other remain linked to concepts trough the sense they provide on the mental plane.

An infinity of sets could be born from *1*; each set from this infinity of sets may containn a infinity of elements, every element belonging to the set of this infinity may contain an infinity of elements, all this desintegration in infinity being processual and potential. But anything starts from *1* or is included in *1*. *It is obvious that *1* is not a set having the significance of the set concept but it is *1*, the matrix of all sets.*

That is why the SAS as a paradox, philosophically implies a deeper reality towards which we are led by the dynamic logic having itself an ontological content.

All last reflects an ontological reality and is why the SAS ---> *1*. Reflected to the R. E. ontological model the paradox of the theory of sets is explainable. It is paradox but only related to a restricted ontological model, that is at bearing of the close universe without orthoexistence. *The paradox reflects a limit of our thinking system only within a certain ontological framewark.*

The paradox is the signal of a deeper reality, reflecting the neccesity of changing our thinking framework. In a way the paradox is part of that old system of thinking , but , it is, at the same time, an opening of this system.

All concepts, are linked to the informatter through their sense, they are all in this unity. From such a point of view all these concepts can only detach from this unity and so, all this multitude of concepts and conceptual objects which may exist (discovered or created) are linked to the informatter (concerning the multitude of senses). That is why the set of all sets is *1*, not because it is limited by *1* but due to the fact that detaching within *1* may take place in multiple infinities. The *1* has an ontological character being the *symbol* of a material reality. *Mathematics is ontological in its fundamentals.*

A set may naturally detach from *1* or it may be mentally detached:

$$*1* \text{ ^^^> } S \qquad (2)$$

but any element of this set may detach (^^^> is the symbol for this process) forming another set and so on:

$$*1* \text{ ^^^> } S \text{ ^^^> } S' \text{ ^^^> } S'' \ldots, \qquad (3)$$

where (S) is the set of the sets detached from *1*, (S') the set of the sets detached from (S), etc. Potentially, each of these sets may have an infinity of elements and each element may desintegrate into an infinity of elements.

The *1* itself is contradictory: it is at the same time a unity and a multipicity. This deep contradiction appears as the SAS paradox. Octav Onicescu was right to affirm that we should not be affraid of paradoxes.

The considerations of this paragraph and of the preceeding paragraphs sustain the idea that the theory of sets and all mathematics are born within an ontological space-temporal and informaterial context (see problem **b** presented at the beginning of this paper). Such a point of view also gives the possibility for a orthophsics of the matter depths because the processes of types (2) and (3) in the informatter may be the support for generating a universe. That is why mathematics may mostly explain the material world and at the same time we may understand why it could be born and develops in the human mind. In spite of this fact mathematics cannot explain anything. It cannot explain just the passing *1* ^^^> (S) because what is born in the matter depths and sometimes mostly on the mental-psychological plane, implying also the matter depths, is of phenomenological, non-mathematizable nature. *Mathematics begins after the set S has bean formed.* It does not mean that sets cannot be formally constituted but not the whole material and conceptual reality may be formally explained. The phenomenological, non-formal aspect usualy lays at the origin of things but as soon as this aspect manifests itself, it has already generated the formal. The blending between the non- formal and the formal in the matter depths, in all that is alive, in human mind and in society is in fact a fundamental reality (problem **e** presented at the beginning of this paper). That is why any formal system belonging to the complexity of the material world or which could reflect part of this world or a conceptual world, cannot be closed but only open, ultimately by the non- formal, phenomenological aspects (problem **e** presented at the beginning of this paper). And the phenomenological aspect is material, mose precisely, informaterial.

If mathematics has so deep ontological bases, if *1* represents the deep reality, what is born in this deep reality is not, first of all, a space but *the set*. The *1* is not a space but deep reality within which by multiplication in sets, spaces are generated too: both the usual space of a universe and other topological spaces attached to it, as well as the abstract spaces but having an ontological background, generated by the human mind.

Ontologically, the space is not previous to the set, but it is true that birth of a set means birth of a space. The set is primordial but the space coexists with the set, from where the never ending problem of the space and set priority is generated (problem **d** presented at the beginning of this paper). In fact this is rather a problem of orthophysics than of mathematics.

FORMAL UNIVERSE, MENTAL UNITY, ONTOLOGICAL UNITY

Any set is selected from elements (which may eventually be sets too) by a belonging relation:

$$x \in A \tag{4}$$

when x belongs to the A set.

The theory of sets uses a minimum number of primary, logical signs ($\neg$ the negation, $\vee$ the disjunction, $\exists$ x the existential quantifier, $\forall$ x the universal quantifier) to which two specific signs are added (= the equality and $\in$ the belonging), all the other being derived from this ones.

The belonging is sufficient to define a set but, usualy, it is linked by a certain property of the elements of a set which, within the theory of sets is expressed by a *relation*.

In this case,

$$x \in A \mid \mathbf{R} \tag{5}$$

where x belongs to A if the relation **R**, refering to x is true.

The belonging is sufficient to define the set.

Thus,

$$x \in A \mid x \in A \qquad (6)$$

in which $x \in A$ is just the relation defining the set.

Any set may be selected by belonging or by a relation which scientifically propers the belonging or it may be constituted, bringing in the set field, those elements which satisfyes that relation defining a set [48].

Where are the elements, necessary to form (to select, to constitute) a set, taken from?

In the formal theory of sets the elements are just *terms* (letters such a, b... or letters combinations {a, b, c...}) and the *relations* are combinations of letters and symbols ($x = y$, $x \in A$). Intuitively the terms are objects (real objects, concepts, symbols) and the relations are links between objects, interations among themselves or even properties of the objects which always could be expressed by the relations of respective objects with those qualities defining these properties (the white objects are in relation with the "white object" belonging to the colour class).

What is the method used when we conceive or make a set?. Either we have a "field of elements" at our disposal or we make them appear first on the mental plane - these are sources of elements necesary to constitute a set. But where from do we draw them out on the mental plane? Is there a given universe of all elements and the human mind only illuminates parts of it to render them evident?

This would mean just platorism.

Is it not normal to think that human mind has power of creation and it brings new elements using a raw material which "supports" the creation of these elements?

This there is a framework of new element generation, namely the mental framework opening an infinite potential for creating elements and sets.

The mental framework appears as the *entity* within which all sets are constituted and from these sets all other sets detach.

The sets are formed related to the concrete reality when we apply the human mind on it; when the mind generates sets, according to the way it implements them in the concrete reality or on the paper, it cannot exceed its framework unity.

In mind, at the origin, any creation of sets is in fact a detachament from the mental framework unit.

All the sets and especially all the sets which could be created, are located in the mind unit.

The SAS is nothing else but the mental unity,

$$SAS \dashrightarrow {}^{*}1^{*}_{mental} \qquad (7)$$

The SAS is a dynamic process which develops in the mental unit which cannot exceed this unity.

If we imagine ${}^{*}1^{*}_{mental}$ as a non-dimensional, non-metric space, then taking into acount the intro-opening of the organism in the informatter, this leads to:

$${}^{*}1^{*}_{mental} \subset {}^{*}1^{*} \qquad (8)$$

meaning that the mental unity, through its informaterial component, is part of the absolute informaterial unit, in its turn, may be also imagined as a non-metric space. The simple image of a non-metric space is not enough for the informatter because each part (element, point)of this "space" is the "neighbour" for any part from the point of view of the fundamental orthosense "to exist" within a full unity. The vicinities change related to other orthosenses and, in this case, more spaces can be generated, according to various types of vicinities. That is why not only,

$${}^{*}1^{*}_{mental} \cup {}^{*}1^{*} \qquad (9)$$

but, in a certain way

$${}^{*}1^{*}_{mental} = {}^{*}1^{*} \qquad (10)$$

In the expression (8), (9) and (10) the ${}^{*}1^{*}_{mental}$ is the informaterial component of the mental unit.

The mental unit is not dimensional, that is why the elements of the sets it concieves may be infinite, any element may be an infinite set too, etc. The ${}^{*}1^{*}_{mental}$ may desintegrate into an infinity of elements and each of these elements may inself be a set with an infinity of elements and so forth.

The mental owns the potential capacity not only for covering the world but also the one of creating, always within the same ontological unit *1*.

From all the axiomatic theories of sets (the system Zermelo- Fraenkel (ZF), Russel-Whitehead, Quine,etc.) only the GB system (von Neumann-Gödel -Bernays) is closer to the above point of view but this does not coincide with it.

The GB theory makes a distinction between classes and sets, fact which seems unnatural, because both classes and sets have elements belonging to them. Cantor, the creator of the theory of sets, did not make any distinction between classes and sets considering that the "belonging" and "set" notions are fundamental concepts of science.

Let us approach the GB theory. If a distinction between classes and sets is made, SAS is no more a set but a class. In this case we may speak, without any logical contradiction, about the class of all sets. In this theory any set is a class but not any class is set.

The class made of all sets, M, is named "universe". The "universe" class is not a set. Any element of the class M is a set.

In other words all the sets are contained in a *universe*, that is something which might coincide with *1* $_{mental}$ or with *1*. There is, in spite of these, a difference. In the GB theory the *universe* is a mathematical, formal concept while *1* $_{mental}$ (or *1*) is a reality.

One may also say that the M reflects the reality *1* $_{mental}$ or *1* on the mathematical plane. To prove this we shall approach some of the properties of the M universe :

a) M cannot be an element of any class. There is not, therefore, any superior class to include it.

b) any class (no matter it is or it is not a set) is included in M.

c) M is a $\in$ -saturated class, that is a class containing beside an y element all the x elements belonging to y. The universe M includes thus all the primary elements and all the classes and, of course, the sets made of them.

d) P (M) = M , where P(M) means the subset class of M. The equality shows that *all* the subset belonging to the M form the M. It is obvious that the P(M) $\subset$ M because any class is included in the M (the above mentioned property b), but is also demonstrated that M $\subset$ P(M), that is P(M) =M.

e) $\cup$ M = M, where $\cup$ M represents the union of the sets existing in the M.

f) $\cap$ M = $\varnothing$, that is the intersection of the sets of the universe is the empty set.

g) $\cap$ $\varnothing$ = M, the intersection of the empty set is the universe.

h) The carthesian product M x M is a class named the total relation.

The carthesian product M x M, as a particular case of the product of two classes raises certain problems.

Being a class it should be included in the M (according to property b). Thus, starting from M and creating new structures such as M x M, they should be included in the M but, in fact, we have a new M and use again M x M, etc. that is, multiplying universe by itself means exceeding it with a more comprehensive universe, that is with an universe which should contain all the *succesive* carthesian products, fact which reminds us the SAS paradox.

The formal universe M may be neither a class, nor a set. The formal universe may not be obeyed to the carthesian product M x M and provided that we cannot operate whith it as we could with a class, why should we make any distinction between the class and the set?. The fact that the difference between the class and the set appeared from the very beginning as unnatural is confirmed like this.

The universe M, being neither a class, nor a set but because the need for its existence is still necesary it has another kind of properties than the class or the set. It is no longer a formal universe. The universe M can be corectly interpreted, from our point of view, only as the mental entity $*1*_{mental}$ or as the ontological entity *1*.

What is important (in fact $*1*_{mental}$ and *1* are not without link because the informatter is the mental bearing substrate) is that *1* (or $*1*_{mental}$) disintegrates in sets, according to the following rules:

$$*1* \text{ -> } [*1*] \qquad (11)$$

$$[*1*] \text{ ^^^^ > } [a, b, c, ...] \qquad (12)$$

$$[a] \text{ ^^^^> } [a1, a2, a3, ...] \qquad (13)$$

$$[a1] \text{ ^^^^ > } [a11, a12, a13, ...] \qquad (14)$$

where the symbol ^^^^ > means disintegration within set.

At (11), the *1* is looked upon abstractly, as being the set [*1*], with (12) this set disintegrates within a set a finite or an infinite number of elements, at (13) an element belonging to the set which first disintegrated may disintegrate in a finite or an infinite set, etc.

The disintegration always takes place from something as if this "something" would receive more of the structure. This disintegration is not merely formal but it reflects a mental or an ontological process.

To operate with the sets from *1* formally, that is to define new sets through belonging or relations, to make reunions, intersections, carthesian products, etc. all these are processings in the framework determined by *1*. And the *1* could not announce its presence except for one of the formal operations announces the exitence of *1*.

This announcement is provided by the formal SAS operation. That is why, *the formal logic, the mathematical logic and mathematics, in general, cannot close in themeselves, they have an opening towards an ontlogical, deeper reality*. Any formal effort of closing it will fail, this *being a reflection of the fact that nothing formal may operate without any support* and this must be manifested by what we name the opening of the formal theory.

The paradox of the theory of sets represents the link between the formal and the non-formal and the fact the formal cannot break off with the mental-psychological and the ontological non-formal. *The paradox does not overthrow the formal theory, on the contrary, it integrates this theory in a larger vision*; it is no longer a paradox except for the strict plane of the formal theory and only at the junction between it and the unavoidable reality which is its source.

Looking upon the formal and the non-formal in their natural relationship, the paradox disappears.

The formal theories which by the so called formal surgery try to escape from the SAS paradox, they run away from the vivid reality of our existence. Mathematics and the human mind cannot detach from the ontological. Both are organically linked by the deep material reality and by space-temporal reality existing in our universe that is by the entire existence. In case mathematics (and the human mind) intends to look upon the formal in itself, a formal ontology is created, more percisely, an informatic ontology which could act within the substance, the matter that is also linked by the proper ontology.

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45 Ibidem, p. 195.

46 Mihai Draganescu, *The Depths of Existence*, op.cit edition, p. 230; 258; 53; 69; 72 and so on (see () at the note 2).

47 Ibidem, p. 195 (see () at the note 2).

48 M.Bourbaki, op.cit , p. E II. 1 - E. II. 7.

49 According to Mircea Reghis, op.cit .